CONCERNING THE GENERAL EQUATIONS OF TURBULENCE

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General equations of turbulent flow are derived for a fluid with large gradients of drift velocities.

1. In order to describe the turbulent flow of a liquid or a gas, one usually begins with the Navier–Stokes equations. We note, however, that these equations have a definite range of applicability and cannot always be used as a basis for deriving the equations of turbulence. This is the case, for example, when the gradients of drift velocities of molecules in a liquid or a gaseous stream vary appreciably over the characteristic hydrodynamic dimension of the container. It then becomes necessary to modify the equations of hydrodynamics. Such a generalization was made by Predvoditelev [1]. The equations of hydrodynamics which he has derived describe such complex hydrodynamic situations as may arise, for example, in a stream of rarefied gas, in a stream near surfaces, and in stellar systems.

The system of hydrodynamic equations which has been derived in [1] for a viscous incompressible fluid is

$$\rho\left[\frac{\partial v_\alpha}{\partial t}+(1-\beta)v_\gamma\frac{\partial v_\alpha}{\partial x^\gamma}\right]=-\frac{\partial p}{\partial x^\alpha}+\mu\frac{\partial^2 v_\alpha}{\partial x^\gamma\partial x^\gamma},\tag{1}$$

and the continuity equation does not change:

$$\frac{\partial v_\gamma}{\partial x^\gamma}=0.\tag{2}$$

Parameter β in (1), which characterizes the deviation from ideal continuity, can be expressed in terms of the Knudsen number and the Mach number

$$|\beta|=\frac{3}{2}\mathrm{Kn}\,\mathrm{M}.\tag{3}$$

Starting from the general equations of hydrodynamics (1) and (2), we will derive the equations of turbulent fluid flow according to Reynolds [2, 3].

2. We will consider [2, 3] the component u of instantaneous velocity to consist of the average velocity component $\bar{u}$ and the turbulent pulsating velocity component u', so that

$$u=\bar{u}+u',\quad v=\bar{v}+v',\quad w=\bar{w}+w'.\tag{4}$$

Then, averaging in accordance with Reynolds' rules, we obtain a system of Reynolds equations for the general equations of hydrodynamics:

$$\rho\left[\frac{\partial\bar{u}}{\partial t}+(1-\beta)\left(\bar{u}\frac{\partial\bar{u}}{\partial x}+\bar{v}\frac{\partial\bar{u}}{\partial y}+\bar{w}\frac{\partial\bar{u}}{\partial z}\right)\right]=-\frac{\partial p}{\partial x}+\mu\Delta\bar{u}$$
$$+(1-\beta)\left[\frac{\partial}{\partial x}(-\rho\overline{u'^2})+\frac{\partial}{\partial y}(-\rho\overline{u'v'})+\frac{\partial}{\partial z}(-\rho\overline{u'w'})\right],$$

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$$\rho\left[\frac{\partial \bar v}{\partial t}+(1-\beta)\left(\bar u\frac{\partial \bar v}{\partial x}+\bar v\frac{\partial \bar v}{\partial y}+\bar w\frac{\partial \bar v}{\partial z}\right)\right]=-\frac{\partial p}{\partial y}+\mu\Delta\bar v+(1-\beta)\left[\frac{\partial}{\partial x}(-\rho\overline{u'v'})\right.$$

$$\left.+\frac{\partial}{\partial y}(-\rho\overline{v'^2})+\frac{\partial}{\partial z}(-\rho\overline{w'v'})\right], \tag{5}$$

$$\rho\left[\frac{\partial \bar w}{\partial t}+(1-\beta)\left(\bar u\frac{\partial \bar w}{\partial x}+\bar v\frac{\partial \bar w}{\partial y}+\bar w\frac{\partial \bar w}{\partial z}\right)\right]$$

$$=-\frac{\partial p}{\partial z}+\mu\Delta\bar w+(1-\beta)\left[\frac{\partial(-\rho\overline{u'w'})}{\partial x}+\frac{\partial(-\rho\overline{v'w'})}{\partial y}+\frac{\partial(-\rho\overline{w'^2})}{\partial z}\right],$$

$$\frac{\partial \bar u}{\partial x}+\frac{\partial \bar v}{\partial y}+\frac{\partial \bar w}{\partial z}=0. \tag{6}$$

Letting parameter β be equal to zero in (5), we obtain the classical Reynolds system of turbulent-flow equations based on the Navier–Stokes equations:

$$\rho\left(\frac{\partial \bar u}{\partial t}+\bar u\frac{\partial \bar u}{\partial x}+\bar v\frac{\partial \bar u}{\partial y}+\bar w\frac{\partial \bar u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\Delta\bar u+\left[\frac{\partial}{\partial x}(-\rho\overline{u'^2})+\frac{\partial}{\partial y}(-\rho\overline{u'v'})+\frac{\partial}{\partial z}(-\rho\overline{u'w'})\right],$$

$$\rho\left(\frac{\partial \bar v}{\partial t}+\bar u\frac{\partial \bar v}{\partial x}+\bar v\frac{\partial \bar v}{\partial y}+\bar w\frac{\partial \bar v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\mu\Delta\bar v+\left[\frac{\partial}{\partial x}(-\rho\overline{u'v'})+\frac{\partial}{\partial y}(-\rho\overline{v'^2})+\frac{\partial}{\partial z}(-\rho\overline{w'v'})\right], \tag{7}$$

$$\rho\left(\frac{\partial \bar w}{\partial t}+\bar u\frac{\partial \bar w}{\partial x}+\bar v\frac{\partial \bar w}{\partial y}+\bar w\frac{\partial \bar w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\Delta\bar w+\left[\frac{\partial}{\partial x}(-\rho\overline{u'w'})+\frac{\partial}{\partial y}(-\rho\overline{v'w'})+\frac{\partial}{\partial z}(-\rho\overline{w'^2})\right].$$

A comparison between systems (5) and (7) shows that the difference between the right-hand sides of the turbulent-flow equations derived here and those the classical equations consists of a change in the Reynolds tensor of turbulent stresses. The additional components of this tensor are proportional to the parameter β. In the applications mentioned here these additional components may be quite significant. The problem of closing system (5), (6) is complex and calls for a separate analysis.

LITERATURE CITED

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